INVESTIGATION OF TEMPERATURE DISTRIBUTION IN A PIECEWISE-HOMOGENEOUS SEAM WITH PRESSED-IN HOT FLUID

G. A. Gamidov, K. I. Kuliev, and I. A. Nasrullaev

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Temperature distribution in a piecewise-homogeneous finite seam exposed to hot fluid and the effect of inhomogeneity in the permeability and thermal properties of separate zones of a seam on the redistribution of temperature fields for flat parallel or axial flow of a pressed-in hot liquid are studied. The differential equations which describe the process of temperature distribution in accordance with [1] are solved for various initial and boundary conditions. Exact analytic formulas are obtained which are useful in numerical computations. The problem under consideration is related to important engineering problems in hydrology, geothermy, as well as in the development of oil or gas fields [2-4, 5].

It is known that oil-bearing seams are not uniform hydrologically or thermally. The publications [3, 4, 6] and other publications deal with thermal processes in such seams, some of the former being of approximate nature. In [4, 6] temperature distribution is studied in a semi-infinite seam which consists of two zones with different, though constant, hydrodynamic and thermal parameters with pressed-in hot fluid in the case of flat parallel flows.

It is noted that formulas for temperature distribution in a seam were found in [6] either for small or large values of time.

In view of its practical value the problem considered in [4, 6] is solved here in the case of the pressed-in hot fluid being filtered in a finite seam, that is, of the oil being withdrawn from the seam at a finite distance from the tunnels.

1. The Flat Parallel Case

Into a finite seam divided into two zones of different permeability and thermal properties let a fluid be swayed in through a straight-line tunnel, the fluid being of temperature T_Γ. The remaining assumptions are the same as in [6]. The finding of the temperature redistribution of the pressed-in fluid for the two zones is mathematically equivalent to the solving of the following system of differential equations:

$$\frac{\partial^2 u_i}{\partial x^2} - \alpha_i \frac{\partial u_i}{\partial x} = \frac{a_1}{a_i} \frac{\partial u_i}{\partial t}, \tag{1.1}$$

$$i = \begin{cases} 1, & 0 \leqslant x \leqslant l \\ 2, & l \leqslant x \leqslant 1 \end{cases}$$

under the initial and boundary conditions,

$$u_1(0, t) = f_1(t);\ u_2(1, t) = f_2(t);\ u_1(l, t) = u_2(l, t); \tag{1.2}$$
$$\frac{\partial u_1(l, t)}{\partial l} = \lambda \frac{\partial u_2(l, t)}{\partial l};$$
$$u_1(x, 0) = F_1(x);\ u_2(x, 0) = F_2(x),$$

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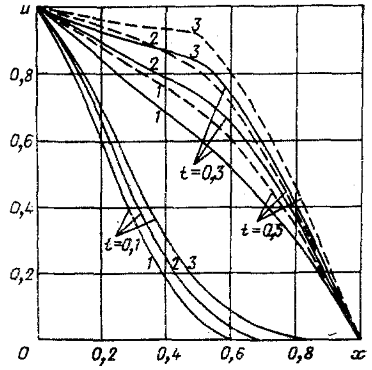

Fig. 1

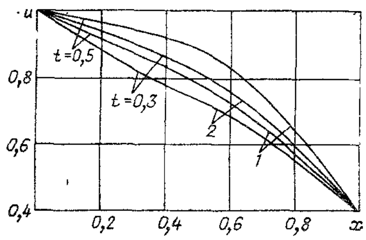

Fig. 2

where

$$u_i = \frac{T_i - T_0}{T_\Gamma - T_0};\; x = \frac{X}{L};\; t = \frac{a_1\tau}{L^2};\; \lambda = \frac{\lambda_2}{\lambda_1};$$

$$\alpha_i = \frac{V_i l}{a_1};\; l = \frac{l_1}{L};$$

L is the seam length; l_1 is the length of the first zone; λ_i, a_i are the coefficients of heat conduction and temperature conductivity of the zones; V_i is the convection rate; X is the dimension coordinate; T_0 is the initial temperature.

The problem (1.1), (1.2) is solved by using the Fourier method with the Duhamel theorem; one obtains

$$u(x,t) = v^{(1)}(x) f_1(t) + v^{(2)}(x) f_2(t) + \tag{1.3}$$

$$+ \sum_{n=1}^{\infty} \left\{ A_n^{(1)} \left[f_1(t) - \delta_n^2 e^{-\delta_n^2 t} \int_0^1 f_1(\tau) e^{\delta_n^2 \tau} d\tau \right] + A_n^{(2)} \left[f_2(t) - \delta_n^2 e^{-\delta_n^2 t} \int_0^1 f_2(\tau) e^{\delta_n^2 \tau} d\tau \right] - A_n^{(3)} e^{-\delta_n^2 t} \right\} X_n(x),$$

where

$$v^{(1)}(x), v^{(2)}(x), X_n(x) = \begin{cases} v_1^{(1)}(x), v_1^{(2)}(x), X_n^{(1)}(x), & 0 \leqslant x \leqslant l, \\ v_2^{(1)}(x), v_2^{(2)}(x), X_n^{(2)}(x), & l \leqslant x \leqslant 1; \end{cases}$$

$$v_1^{(1)}(x) = 1 + \lambda N_1 \frac{\alpha_2}{\alpha_1} e^{(\alpha_2 - \alpha_1)l} \left(1 - e^{\alpha_1 x}\right);$$

$$v_2^{(1)}(x) = 1 - v_1^{(1)}(x);$$

$$v_1^{(2)}(x) = N_1 \left(e^{\alpha_2} - e^{\alpha_2 x}\right);$$

$$v_2^{(2)}(x) = 1 + N_1 \lambda \frac{\alpha_2}{\alpha_1} e^{(\alpha_2 - \alpha_1)} \left(e^{\alpha_2 x} - 1\right);$$

$$v_1^{(3)}(x) = F_1(x);\; v_2^{(3)}(x) = F_2(x);$$

$$X_n^{(1)}(x) = \frac{\sin \beta_1 x}{\sin \beta_1 l} e^{-\frac{\alpha_1}{2}(l-x)};\; X_n^{(2)}(x) = \frac{\sin \beta_2 (1-x)}{\sin \beta_2 (1-l)} e^{-\frac{\alpha_2}{2}(x-l)};$$

$$N_1 = \left[\lambda \frac{\alpha_2}{\alpha_1} e^{(\alpha_2 - \alpha_1)l} \left(e^{\alpha_1 l} - 1\right) - e^{\alpha_2 l} + e^{\alpha_2}\right]^{-1};$$

$$\beta_{1,2} = \frac{\alpha_1}{2} \pm \sqrt{\delta_n^2 - \frac{\alpha_1^2}{4}};$$

$$A_n^{(j)} = -M_n \left[\lambda_1 e^{\alpha_1 l} \int_0^l e^{-\alpha_1 x} v_1^{(j)}(x) X_n^{(1)}(x)\, dx + \lambda_2 e^{\alpha_2 l} \int_l^1 e^{-\alpha_2 x} v_2^{(j)}(x) X_n^{(2)}(x)\, dx\right];$$

$$M_n = \left[\lambda_1 e^{\alpha_1 l} \int_0^l e^{-\alpha_1 x} X^{(1)}(x)\,dx + \lambda_2 e^{\alpha_2 l} \int_l^1 e^{-\alpha_2 x} X_n^{(2)^2}(x)\,dx\right]^{-1},$$

and δ_n are the roots of the transcendental equation

$$\beta_1 \operatorname{ctg} \beta_1 l - \lambda\beta_2 \operatorname{ctg} \beta_2 (1 - l) = \lambda \frac{\alpha_2}{2} - \frac{\alpha_1}{2}. \tag{1.4}$$

In particular, if the initial and boundary conditions are constant, that is, for

$$u_1(0, t) = \overline{T}_c;\ u_2(1, t) = \overline{T}_K;\ u_1(x, 0) = u_2(x, 0) = 0,$$

the solution (1.3) becomes

$$u = \overline{T}_c v^{(1)}(x) + \overline{T}_K v^{(2)}(x) + \sum_{n=1}^{\infty} \left(\overline{T}_K A_n^{(1)} + \overline{T}_c A_n^{(2)}\right) X_n(x)\, e^{-\delta_n^2 t}. \tag{1.5}$$

Of course, by using (1.5) the computations do not present any special difficulties since the series converges rapidly.

2. The Axially Symmetric Case

For this case the hot liquid sways into a circular seam consisting of two concentric zones of different constant thermal parameters. Then the temperature function u(r, t) satisfies the following differential equations:

$$\frac{\partial^2 u_i}{\partial r^2} + \frac{1 - 2\nu_i}{r} \frac{\partial u_i}{\partial r} = \frac{a_1}{a_i} \frac{\partial u_i}{\partial t}, \quad i = 1, 2 \tag{2.1}$$

as well as the initial and boundary conditions

$$\begin{gathered} u_1(R_c, t) = f_1(t);\ u_2(1, t) = f_2(t);\ u_1(R, t) = u_2(R, t); \\ \frac{\partial u_1(R, t)}{\partial R} = \lambda \frac{\partial u_2(R, t)}{\partial R}; \\ u_1(r, 0) = F_1(r);\ u_2(r, 0) = F_2(r), \end{gathered} \tag{2.2}$$

where

$$r = \frac{r_1}{R_p};\ R_c = \frac{r_h}{R_p};\ t = \frac{a_1 \tau}{R_p^2};$$

$$\nu_i = \frac{QC_i\rho_i}{2\pi m h \lambda_i};\ R = \frac{R_1}{R_p};\ i = 1, 2,$$

and Q is the expenditure of the pressed-in fluid; C_i, ρ_i are the heat capacities and densities of the seam zones; m is porosity; h is depth; r_h is the hole radius; R_p is the radius of the seam profile; R_1 is the radius of the zone boundary; r_1, t are the coordinates.

The solution of the problem (2.1), (2.2) is given by

$$\begin{aligned} u(r, t) = v^{(1)}(r) f_1(t) + v^{(2)}(r) f_2(t) + \sum_{n=1}^{\infty} \Big\{ A_n^{(1)} \Big[f_1(t) - \delta_n^2 e^{-\delta_n^2 t} \times \\ \times \int_0^t f_1(\tau) e^{\delta_n^2 \tau} d\tau \Big] + A_n^{(2)} \Big[f_2(t) - \delta_n^2 e^{-\delta_n^2 t} \int_0^t f_2(\tau) e^{\delta_n^2 \tau} d\tau \Big] - A_n^{(3)} e^{-\delta_n^2 t} \Big\} \varphi_n(r) \end{aligned} \tag{2.3}$$

where

$$v^{(1)}(r),\ v^{(2)}(r),\ \varphi_n(r) = \begin{cases} v_1^{(1)}(r),\ v_1^{(2)}(r),\ \varphi_n^{(1)}(r); & R_c \leqslant r \leqslant R, \\ v_2^{(1)}(r),\ v_2^{(2)}(r),\ \varphi_n^{(2)}(r), & R \leqslant r \leqslant 1; \end{cases}$$

$$v_1^{(1)}(r) = 1 - v_2^{(1)}(r); \quad v_1^{(2)}(r) = \frac{N_2}{2\nu_2}\left(r^{2\nu_2} - 1\right);$$

$$v_2^{(2)}(r) = 1 - v_1^{(2)}(r); \quad v_2^{(1)}(r) = -N_2 \frac{\lambda}{2\nu_1} R^{2(\nu_2-\nu_1)}\left(r^{2\nu_1} - R_C^{2\nu_1}\right);$$

$$v_2^{(3)}(r) = F_1(r); \; v_1^{(3)}(r) = F_2(r);$$

$$N_2 = \left[\frac{R^{2\nu_2}-1}{2\nu_2} - \frac{\lambda}{2\nu_1} R^{2(\nu_2-\nu_1)}\left(R^{2\nu_1} - R_C^{2\nu_1}\right)\right]^{-1};$$

$$\varphi_n^{(1)}(r) = \left(\frac{r}{R}\right)^{\nu_1} \frac{I_{\nu_1}(\delta r)\, Y_{\nu_1}(\delta R_C) - I_{\nu_1}(\delta R_C)\, Y_{\nu_1}(\delta r)}{I_{\nu_1}(\delta R)\, Y_{\nu_1}(\delta R_C) - I_{\nu_1}(\delta R_C)\, Y_{\nu_1}(\delta R)};$$

$$\varphi_n^{(2)}(r) = \left(\frac{r}{R}\right)^{\nu_2} \frac{I_{\nu_2}(\bar{\delta} r)\, Y_{\nu_2}(\bar{\delta}) - I_{\nu_2}(\bar{\delta})\, Y_{\nu_2}(\bar{\delta} r)}{I_{\nu_2}(\bar{\delta} R)\, Y_{\nu_2}(\bar{\delta}) - I_{\nu_2}(\bar{\delta})\, Y_{\nu_2}(\bar{\delta} R)}; \bar{\delta} = \frac{a_1}{a_2}\delta,$$

$$A_n^{(j)} = -M_n\left[\lambda_1 R^{2\nu_1-1}\int_{R_C}^{R} r^{1-2\nu_1}\varphi_n^{(1)}(r)\, v_1^{(j)}(r)\, dr + \lambda_2 R^{2\nu_2-1}\int_{R}^{1} r^{1-2\nu_2}\varphi_n^{(2)}(r)\, v_2^{(j)}(r)\, dr\right];$$

$$M_n = \left[\lambda_1 R^{2\nu_1-1}\int_{R_C}^{R} r^{1-2\nu_1}\varphi_n^{(1)^2}(r)\, dr + \lambda_2 R^{2\nu_2-1}\int_{R}^{1} r^{1-2\nu_2}\varphi_n^{(2)^2}(r)\, dr\right]^{-1},$$

$$j = 1, 2, 3.$$

In the above $I\nu(x)$ and $Y\nu(x)$ denote Bessel functions of real argument of the ν-th order of the first or second kind, respectively;

δ_n are the roots of the equation

$$a_2 \frac{I_{\nu_1-1}(\delta R)\, Y_{\nu_1}(\delta R_C) - I_{\nu_1}(\delta R_C)\, Y_{\nu_1-1}(\delta R)}{I_{\nu_1}(\delta R)\, Y_{\nu_1}(\delta R_C) - I_{\nu_1}(\delta R_C)\, Y_{\nu_1}(\delta R)}$$
$$= a_1 \frac{I_{\nu_2-1}(\bar{\delta} R)\, Y_{\nu_2}(\bar{\delta}) - I_{\nu_2}(\bar{\delta})\, Y_{\nu_2-1}(\bar{\delta} R)}{I_{\nu_2}(\bar{\delta} R)\, Y_{\nu_2}(\bar{\delta}) - I_{\nu_2}(\bar{\delta})\, Y_{\nu_2}(\bar{\delta} R)}.$$

In the case of constant boundary conditions,

$$u_1(r_c, t) = u_c; \; u(1, t) = u_K;$$
$$u_1(r, 0) = u_2(r, 0) = 0,$$

the solution (2.3) assumes a simple form,

$$u = u_c v^{(1)}(r) + u_K v^{(2)}(r) + \sum_{n=1}^{\infty}\left(u_c A_n^{(1)} + u_K A_n^{(2)}\right)\varphi_n(r)\, e^{-\delta_n^2 t}.$$

To study the effect of the seam inhomogeneity on the temperature field computations were carried out using the formula (1.5) for h = 10 m; $\alpha_1 = 1$; $\alpha_2 = 1; 2$; $\lambda = 0.2; 0.5; 1.0$; $l = 0.5$; $\overline{T}_k = 0; 0.4$.

The following simple asymptotic expansions are then found for the roots of Eq. (1.4):

$$\delta_n^2 = (n\pi)^2 + \frac{1}{2} - \frac{1}{32(n\pi)^2} + \ldots \text{ for } \lambda = \frac{1}{2};$$

$$\delta_n^2 = (n\pi)^2 + \frac{11}{8} - \frac{35}{128(n\pi)^2} + \ldots \text{ for } \lambda = \frac{1}{5}.$$

The results of the computations are shown in Figs. 1 and 2. In the case of $\overline{T}_k = 0$ (Fig. 1) the changing of λ_1 with λ_k kept fixed (the curves 1, 2, 3, correspond to the values of $\lambda = 1; 0.5; 0.2$ respectively) has a considerable effect on the heat distribution in the seam and, in particular, for high values of time. A similar tendency can also be observed in the case of $T_k = 0.4$ (Fig. 2) the only difference being that now the rate of temperature change as a function of x is lower than for the previous case.

In view of the above it follows that when operating near the front face of the hole as well as in the case of swaying-in of the hot fluid into the seam it is necessary that the inhomogeneity of the seam as regards its thermal properties be taken into account.

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